Examples and application of solubility product

أمثلة وتطبيقات حاصل الاذابة

The type of questions about solubility-product constant (Ksp) are limited and all of them related to the sparingly soluble salts in saturated aqueous solutions that are in contact with an excess of non-dissolved salt. The classic problems are of three types:

1. Calculation of Ksp value from the solubility of the salt.
2. Calculation of the solubility of the salt from Ksp.
3. Calculation of one ion concentration of the salt ions in saturated solution if the value of Ksp and the concentration of the other ions are known.

ان طبيعة الاسئلة الخاصة بثابت حاصل الاذابة محدودة وترتبط بالاملاح الشحيحة الذوبان في المحاليل المائية المشبعة والتي تكون في تماس مع الزيادة من الملح غير الذائب. الاسئلة التقليدية تكون على ثلاثة انواع:

1- حساب ثابت حاصل الاذابة من خلال ذوبانية الملح.

2- حساب ذوبانية الملح من معرفة ثابت حاصل الاذابة.

3- حساب تركيز احد ايونات الملح في المحلول المشبع اذا كانت قيمة ثابت حاصل الاذابة وتراكيز الايونات الاخرى معلومة.

1. Calculation of Ksp value from the solubility of the salt:

Ex. 1 If the solubility of AgBr = $2x10^{-5}$ g/100mL at 20°C, calculate Ksp value? (Mwt=187.8 g/mole)

The solubility should be calculated in unit of mole/liter so:

weight	volume
$2x10^{-5}$	100 mL
?	1000 mL

$$\Rightarrow\Rightarrow\Rightarrow \quad ? = \frac{2x10^{-5} \text{ X } 1000}{100} = 2x10^{-4} \text{ g/L}$$

Concentration (mole/L) = concentration (g/L) / Mwt

$$= \frac{2x10^{-4}}{187.8} = 1.065x10^{-6} \text{ mole/L}$$

or

$$M = \frac{wt}{Mwt} x \frac{1000}{VmL} \quad \Rightarrow\Rightarrow\Rightarrow \quad M = \frac{2x10^{-5}}{187.8} x \frac{1000}{100} = 1.065x10^{-6} \text{ mole/L}$$

This means that $1.065x10^{-6}$ mole of AgBr is dissolved in one liter of solution (available as ions). Therefore, each liter of the solution contains:

$[Ag^+] = 1.065x10^{-6}$ mole/L

$[Br^-] = 1.065x10^{-6}$ mole/L

Ksp calculated as follows:

$$AgBr \rightleftharpoons Ag^+ + Br^-$$

$Ksp = [Ag^+][Br^-]$

$= (1.065x10^{-6}$ mole/L) x $(1.065x10^{-6}$ mole/L) [or$(1.065x10^{-6}$ mole/L$)^2$]

$= 1.134x10^{-12}$

Ex. 2 If the solubility of BaF_2 at 20°C is 0.161 g/100 mL in water, calculate Ksp value? (Mwt=175.36g/mole)

The solubility should be calculated in unit of mole/liter so:

weight	volume
0.161	100 mL
?	1000 mL

$\Rightarrow\Rightarrow\Rightarrow$? = $\frac{0.161 \text{ X } 1000}{100}$ = 1.61 g/L

Concentration (mole/L) = concentration (g/L) / Mwt

$= \frac{1.61}{175.36} = 0.00919$ mole/L

$$BaF_2 \rightleftharpoons Ba^{2+} + 2F^-$$

$[BaF_2]$= 0.00919 mole/L $\Rightarrow\Rightarrow\Rightarrow$ $[Ba^{2+}]$ = 0.00919 mole/L

$\Rightarrow\Rightarrow\Rightarrow$ $[F^-]$ = 2 x 0.00919 = 0.01838 mole/L

$Ksp = [Ba^{2+}][F^-]^2$

$= (0.00919) \times (0.01838)^2$

$= 3.09x10^{-6}$

Exercise: If the solubility of $Pb_3(PO_4)_2$ at 20°C is $1.4x10^{-4}$ g/L in water, calculate Ksp value? (Mwt= 811.5427 g/mole)

2. Calculation of the solubility of the salt from Ksp:

Ex. 1 If Ksp value of AgI is $8.3x10^{-17}$ at 20°C, calculate the solubility of AgI in mole/L and in g/100mL? (Mwt=234.8 g/mole)

$$AgI_{(s)} \rightleftharpoons Ag^{+} + I^{-}$$
$$X \qquad X \qquad X$$

(X represent molar concentration)

As indicated from the equation:

$[AgI] = X = [Ag^{+}] = [I^{-}]$

$Ksp = [Ag^{+}][I^{-}] = X.\ X = X^2$

$8.3x10^{-17} = X^2$

$X = \sqrt{8.3x10^{-17}} = 9.11x10^{-9}$ mole/L

Concentration (g/L) = concentration (mole/L) x Mwt

$= 9.11x10^{-9} \times 234.8$

$= 2.14x10^{-6}$ g/L

$$\frac{2.14x10^{-6}\ g/L \times 100}{1000} = 2.14x10^{-7}\ g/100mL$$

Ex. 2 What is the solubility of $Pb(OH)_2$ if Ksp value at 20°C is $2.5x10^{-16}$?

$$Pb(OH)_2 \rightleftharpoons Pb^{2+} + 2OH^{-}$$
$$X \qquad 2X$$

$Ksp = [Pb^{2+}][OH^{-}]^2$

$2.5x10^{-16} = X.\ (2X)^2 = 4X^3$

$X^3 = \frac{2.5x10^{-16}}{4} = 0.625x10^{-16}$

$X = \sqrt[3]{0.625x10^{-16}} = 3.969x10^{-5}$ mole/L

Exercise: What is the solubility of $Pb(IO_3)_2$ if Ksp value at 20°C is $2.5x10^{-13}$?

3. Calculation of one ion concentration of the salt ions in saturated solution on the bases of the concentration of the other ion:

Ex. 1 Calculate the concentration of Ag^+ ion in saturated solution of Ag_2CrO_4 (Ksp = $3.4x10^{-12}$) if you know that the concentration of CrO_4^{2-} ion is $2x10^{-2}$ mole/L?

$$Ag_2CrO_4 \rightleftharpoons 2Ag^+ + CrO_4^{2-}$$

$Ksp = [Ag^+]^2 [CrO_4^{2-}]$

$3.4x10^{-12} = [Ag^+]^2 \times 2x10^{-2}$

$[Ag^+]^2 = \frac{3.4x10^{-12}}{2x10^{-2}} = 1.7x10^{-10}$

$[Ag^+] = \sqrt{1.7x10^{-10}} = 1.194x10^{-5}$ mole/L

Common ion effect تأثير الأيون المشترك

The solubility of an ionic precipitate (an insoluble salt) decreases when a soluble compound containing one of the ions of the precipitate is added to the solution. This behavior is called the **common-ion effect**. The common-ion effect is a mass-action effect predicted from Le Châtelier's principle. This effect leads to decreasing the concentration of the other ion at equilibrium only if the temperature and pressure are constant. Precipitates are more soluble in water than in solutions containing one of the ions of the precipitates.

ان قابلية ذوبان الراسب الأيوني (ملح غير قابل للذوبان) تقل عند إضافة مركب قابل للذوبان (يحتوي على أحد أيونات الراسب) للمحلول. يسمى هذا السلوك بتأثير الأيون المشترك. تأثير الأيون المشترك هو تأثير فعل الكتلة المتوقع من مبدأ لي شاتليه. هذا التأثير يؤدي إلى نقصان تركيز الايون الاخر بشرط ثبوت درجة الحرارة والضغط. الرواسب تكون اكثر ذوبانية في الماء مقارنة بالمحاليل التي تحتوي على الايون المشترك.

For example:

$$NH_4OH \rightleftharpoons \boxed{NH_4^+} + OH^-$$ [adding NH_4Cl will shift the equilibrium to the left]

$$NH_4Cl \rightarrow \boxed{NH_4^+} + Cl^-$$

common-ion

Also:

$$AgI \rightleftharpoons \boxed{Ag^+} + I^-$$ [adding $AgNO_3$ will shift the equilibrium to the left]

$$AgNO_3 \rightarrow \boxed{Ag^+} + NO_3^-$$

common-ion

<u>Ex</u>: Calculate the solubility of CaF_2 ($K_{sp} = 4.0x10^{-11}$) in:

1. Distilled water.
2. Solution of 0.01 M $CaCl_2$.
3. Solution of 0.01 M NaF.

1. Distilled water:

$$CaF_2 \rightleftharpoons \underset{X}{Ca^{2+}} + \underset{2X}{2F^-}$$

$$Ksp = [Ca^{2+}]\,[F^-]^2$$

$$4.0x10^{-11} = X.\,(2X)^2 = 4X^3$$

$$X^3 = \frac{4.0x10^{-11}}{4} = 1.0x10^{-11}$$

$$X = \sqrt[3]{1.0x10^{-11}} = 2.154x10^{-4} \text{ mole/L}$$

2. Solution of 0.01 M $CaCl_2$:

$$CaF_2 \rightleftharpoons \boxed{Ca^{2+}} + 2F^-$$

$$CaCl_2 \rightarrow \boxed{Ca^{2+}} + 2Cl^-$$ ($CaCl_2$ is completely ionized)

common-ion

$[Ca^{2+}] = X + 0.01$

$[F^-] = 2X$

$Ksp = [Ca^{2+}] [F^-]^2 = (X + 0.01) (2X)^2$ $X \ll 0.01 \Rightarrow\Rightarrow\Rightarrow$ تهمل

$4.0x10^{-11} = (0.01) (4X^2) = 0.04 X^2$

$X^2 = 0.1x10^{-10}$

$X = 3.16x10^{-5}$ mole/L

3. Solution of 0.01 M NaF:

$CaF_2 \rightleftharpoons Ca^{2+} + 2F^-$

$NaF \rightarrow Na^+ + F^-$ (NaF is completely ionized)

common-ion

$[Ca^{2+}] = X$

$[F^-] = 2X + 0.01$

$Ksp = [Ca^{2+}] [F^-]^2 = (X) (2X + 0.01)^2$ $2X \ll 0.01 \Rightarrow\Rightarrow\Rightarrow$ تهمل

$4.0x10^{-11} = (X) (0.01)^2 = 1.0x10^{-4} X$

$X = 4.0x10^{-7}$ mole/L