

**Examples and application of solubility product****أمثلة وتطبيقات حاصل الاذابة**

The type of questions about solubility-product constant ( $K_{sp}$ ) are limited and all of them related to the sparingly soluble salts in saturated aqueous solutions that are in contact with an excess of non-dissolved salt. The classic problems are of three types:

1. Calculation of  $K_{sp}$  value from the solubility of the salt.
2. Calculation of the solubility of the salt from  $K_{sp}$ .
3. Calculation of one ion concentration of the salt ions in saturated solution if the value of  $K_{sp}$  and the concentration of the other ions are known.

ان طبيعة الاسئلة الخاصة بثابت حاصل الاذابة محدودة وترتبط بالاملاح الشحيحة الذوبان في المحاليل المائية المشبعة والتي تكون في تماس مع الزيادة من الملح غير الذائب. الاسئلة التقليدية تكون على ثلاثة انواع:

- 1- حساب ثابت حاصل الاذابة من خلال ذوبانية الملح.
- 2- حساب ذوبانية الملح من معرفة ثابت حاصل الاذابة.
- 3- حساب تركيز احد ايونات الملح في المحلول المشبع اذا كانت قيمة ثابت حاصل الاذابة وتراكيز الايونات الاخرى معلومة.

**1. Calculation of  $K_{sp}$  value from the solubility of the salt:**

**Ex. 1** If the solubility of  $\text{AgBr} = 2 \times 10^{-5} \text{ g/100 mL}$  at  $20^\circ\text{C}$ , calculate  $K_{sp}$  value? ( $M_{wt} = 187.8 \text{ g/mole}$ )

The solubility should be calculated in unit of mole/liter so:

weight    volume

$2 \times 10^{-5}$     100 mL

$$? \quad 1000 \text{ mL} \Rightarrow \Rightarrow \Rightarrow ? = \frac{2 \times 10^{-5} \times 1000}{100} = 2 \times 10^{-4} \text{ g/L}$$

$$\begin{aligned} \text{Concentration (mole/L)} &= \text{concentration (g/L)} / M_{wt} \\ &= \frac{2 \times 10^{-4}}{187.8} = 1.065 \times 10^{-6} \text{ mole/L} \end{aligned}$$

or

$$M = \frac{wt}{M_{wt}} \times \frac{1000}{V_{mL}} \Rightarrow \Rightarrow \Rightarrow M = \frac{2 \times 10^{-5}}{187.8} \times \frac{1000}{100} = 1.065 \times 10^{-6} \text{ mole/L}$$

This means that  $1.065 \times 10^{-6}$  mole of AgBr is dissolved in one liter of solution (available as ions). Therefore, each liter of the solution contains:

$$[\text{Ag}^+] = 1.065 \times 10^{-6} \text{ mole/L}$$

$$[\text{Br}^-] = 1.065 \times 10^{-6} \text{ mole/L}$$

K<sub>sp</sub> calculated as follows:



$$K_{sp} = [\text{Ag}^+][\text{Br}^-]$$

$$= (1.065 \times 10^{-6} \text{ mole/L}) \times (1.065 \times 10^{-6} \text{ mole/L}) \quad [\text{or } (1.065 \times 10^{-6} \text{ mole/L})^2]$$

$$= 1.134 \times 10^{-12}$$

**Ex. 2** If the solubility of BaF<sub>2</sub> at 20°C is 0.161 g/100 mL in water, calculate K<sub>sp</sub> value? (Mwt=175.36g/mole)

The solubility should be calculated in unit of mole/liter so:

weight    volume

$$0.161 \quad 100 \text{ mL}$$

$$? \quad 1000 \text{ mL} \Rightarrow \Rightarrow \Rightarrow ? = \frac{0.161 \times 1000}{100} = 1.61 \text{ g/L}$$

$$\text{Concentration (mole/L)} = \text{concentration (g/L)} / \text{Mwt}$$

$$= \frac{1.61}{175.36} = 0.00919 \text{ mole/L}$$



$$[\text{BaF}_2] = 0.00919 \text{ mole/L} \Rightarrow \Rightarrow \Rightarrow [\text{Ba}^{2+}] = 0.00919 \text{ mole/L}$$

$$\Rightarrow \Rightarrow \Rightarrow [\text{F}^-] = 2 \times 0.00919 = 0.01838 \text{ mole/L}$$

$$K_{sp} = [\text{Ba}^{2+}][\text{F}^-]^2$$

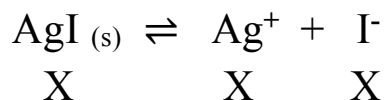
$$= (0.00919) \times (0.01838)^2$$

$$= 3.09 \times 10^{-6}$$

**Exercise:** If the solubility of Pb<sub>3</sub>(PO<sub>4</sub>)<sub>2</sub> at 20°C is  $1.4 \times 10^{-4}$  g/L in water, calculate K<sub>sp</sub> value? (Mwt= 811.5427 g/mole)

**2. Calculation of the solubility of the salt from  $K_{sp}$ :**

**Ex. 1** If  $K_{sp}$  value of AgI is  $8.3 \times 10^{-17}$  at  $20^\circ\text{C}$ , calculate the solubility of AgI in mole/L and in g/100mL? (Mwt=234.8 g/mole)



(X represent molar concentration)

As indicated from the equation:

$$[\text{AgI}] = X = [\text{Ag}^+] = [\text{I}^-]$$

$$K_{sp} = [\text{Ag}^+][\text{I}^-] = X \cdot X = X^2$$

$$8.3 \times 10^{-17} = X^2$$

$$X = \sqrt{8.3 \times 10^{-17}} = 9.11 \times 10^{-9} \text{ mole/L}$$

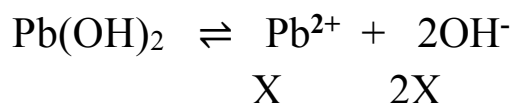
$$\text{Concentration (g/L)} = \text{concentration (mole/L)} \times \text{Mwt}$$

$$= 9.11 \times 10^{-9} \times 234.8$$

$$= 2.14 \times 10^{-6} \text{ g/L}$$

$$\frac{2.14 \times 10^{-6} \text{ g/L} \times 100}{1000} = 2.14 \times 10^{-7} \text{ g/100mL}$$

**Ex. 2** What is the solubility of  $\text{Pb}(\text{OH})_2$  if  $K_{sp}$  value at  $20^\circ\text{C}$  is  $2.5 \times 10^{-16}$  ?



$$K_{sp} = [\text{Pb}^{2+}][\text{OH}^-]^2$$

$$2.5 \times 10^{-16} = X \cdot (2X)^2 = 4X^3$$

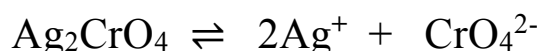
$$X^3 = \frac{2.5 \times 10^{-16}}{4} = 0.625 \times 10^{-16}$$

$$X = \sqrt[3]{0.625 \times 10^{-16}} = 3.969 \times 10^{-5} \text{ mole/L}$$

**Exercise:** What is the solubility of  $\text{Pb}(\text{IO}_3)_2$  if  $K_{sp}$  value at  $20^\circ\text{C}$  is  $2.5 \times 10^{-13}$ ?

### 3. Calculation of one ion concentration of the salt ions in saturated solution on the bases of the concentration of the other ion:

**Ex. 1** Calculate the concentration of  $\text{Ag}^+$  ion in saturated solution of  $\text{Ag}_2\text{CrO}_4$  ( $K_{\text{sp}} = 3.4 \times 10^{-12}$ ) if you know that the concentration of  $\text{CrO}_4^{2-}$  ion is  $2 \times 10^{-2}$  mole/L?



$$K_{\text{sp}} = [\text{Ag}^+]^2 [\text{CrO}_4^{2-}]$$

$$3.4 \times 10^{-12} = [\text{Ag}^+]^2 \times 2 \times 10^{-2}$$

$$[\text{Ag}^+]^2 = \frac{3.4 \times 10^{-12}}{2 \times 10^{-2}} = 1.7 \times 10^{-10}$$

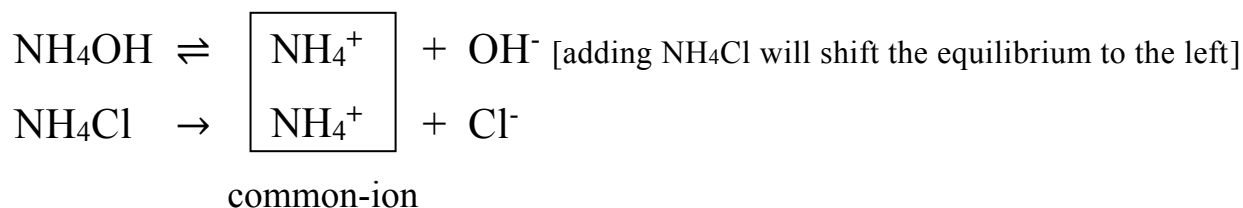
$$[\text{Ag}^+] = \sqrt{1.7 \times 10^{-10}} = 1.194 \times 10^{-5} \text{ mole/L}$$

### Common ion effect      تأثير الأيون المشترك

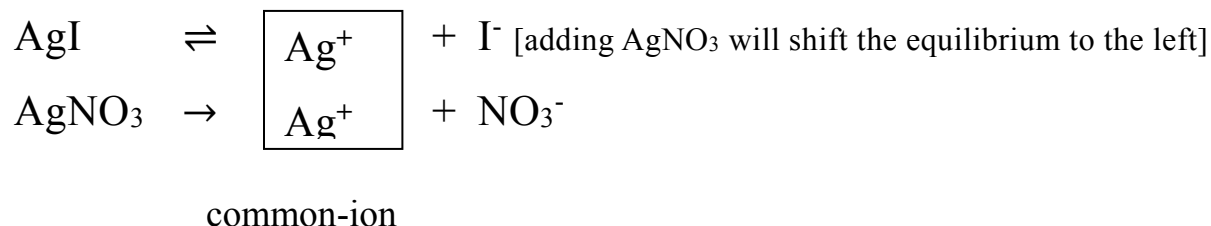
The solubility of an ionic precipitate (an insoluble salt) decreases when a soluble compound containing one of the ions of the precipitate is added to the solution. This behavior is called the **common-ion effect**. The common-ion effect is a mass-action effect predicted from Le Châtelier's principle. This effect leads to decreasing the concentration of the other ion at equilibrium only if the temperature and pressure are constant. Precipitates are more soluble in water than in solutions containing one of the ions of the precipitates.

ان قابلية ذوبان الراسب الأيوني (ملح غير قابل للذوبان) تقل عند إضافة مركب قابل للذوبان (يحتوي على أحد أيونات الراسب) للمحلول. يسمى هذا السلوك بتأثير الأيون المشترك. تأثير الأيون المشترك هو تأثير فعل الكتلة المتوقع من مبدأ لي شاتليه. هذا التأثير يؤدي إلى نقصان تركيز الأيون الآخر بشرط ثبوت درجة الحرارة والضغط. الرواسب تكون أكثر ذوبانية في الماء مقارنة بالمحاليل التي تحتوي على الأيون المشترك.

For example:



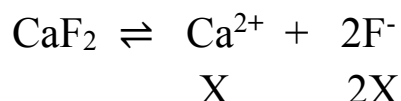
Also:



**Ex:** Calculate the solubility of  $\text{CaF}_2$  ( $K_{\text{sp}} = 4.0 \times 10^{-11}$ ) in:

1. Distilled water.
2. Solution of 0.01 M  $\text{CaCl}_2$ .
3. Solution of 0.01 M  $\text{NaF}$ .

1. Distilled water:



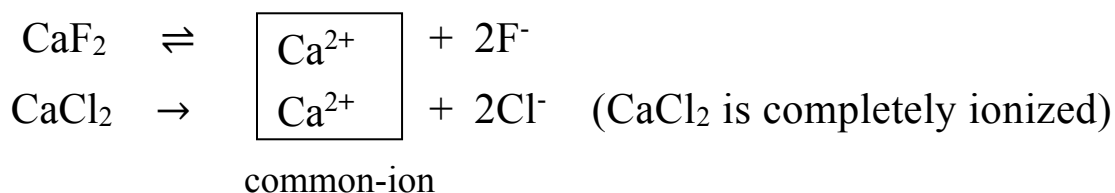
$$K_{\text{sp}} = [\text{Ca}^{2+}] [\text{F}^-]^2$$

$$4.0 \times 10^{-11} = \text{X} \cdot (2\text{X})^2 = 4\text{X}^3$$

$$\text{X}^3 = \frac{4.0 \times 10^{-11}}{4} = 1.0 \times 10^{-11}$$

$$\text{X} = \sqrt[3]{1.0 \times 10^{-11}} = 2.154 \times 10^{-4} \text{ mole/L}$$

2. Solution of 0.01 M  $\text{CaCl}_2$ :



$$[\text{Ca}^{2+}] = X + 0.01$$

$$[\text{F}^-] = 2X$$

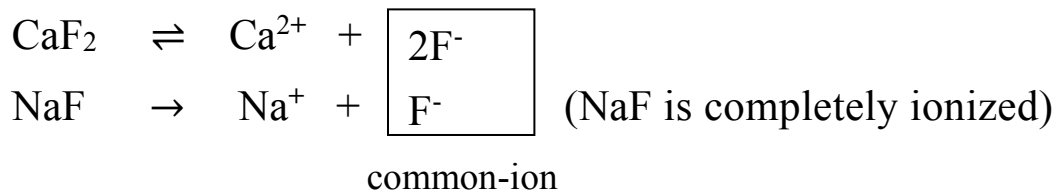
$$K_{sp} = [\text{Ca}^{2+}] [\text{F}^-]^2 = (X + 0.01) (2X)^2 \quad X \ll 0.01 \Rightarrow \Rightarrow \Rightarrow \text{تھمل}$$

$$4.0 \times 10^{-11} = (0.01) (4X^2) = 0.04 X^2$$

$$X^2 = 0.1 \times 10^{-10}$$

$$X = 3.16 \times 10^{-5} \text{ mole/L}$$

3. Solution of 0.01 M NaF:



$$[\text{Ca}^{2+}] = X$$

$$[\text{F}^-] = 2X + 0.01$$

$$K_{sp} = [\text{Ca}^{2+}] [\text{F}^-]^2 = (X) (2X + 0.01)^2 \quad 2X \ll 0.01 \Rightarrow \Rightarrow \Rightarrow \text{تھمل}$$

$$4.0 \times 10^{-11} = (X) (0.01)^2 = 1.0 \times 10^{-4} X$$

$$X = 4.0 \times 10^{-7} \text{ mole/L}$$